Measuring Issue-Specific Preferences from Votes

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Motivation

Two challenges and existing approaches:

- How can we measure issue-specific ideal points?
  - Gerrish and Blei's (2012) model which requires text data;
  - One-dimensional model with only a subset of voting data.
- The interpretation of latent dimensions in a multidimensional policy space is unclear.
  - E.g., DW-Nominate labels the first dimension as "economic" and the second one as "other votes."

I propose a new model that estimates issue-specific hyperplane and ideal points.

Setup

Data:

- roll-call votes ($y_{ij} = 1$ if the ith legislator votes Yea on the jth roll call and 0 otherwise);
- issue indicator of each bill (z_j for each j).

Issue-Specific IRT Model

Data Generating Process:

$$\boldsymbol{u}_j \sim \textbf{vMF}(\theta_{z_j}, \rho) \quad (1)$$

$$\Pr(y_{ij} = 1) = \Phi(w_j \boldsymbol{u}_j^\top \boldsymbol{x}_i - \alpha_j) \quad (2)$$

where $\textbf{vMF}(\cdot)$ denotes von Mises-Fisher distribution and ...

- θ_z: *direction* of issue-specific hyperplane (i.e., $||\theta_z|| = 1$);
- $\boldsymbol{u}_j$: *direction* of the jth hyperplane (i.e., $||\boldsymbol{u}_j|| = 1$);
  - which known as *discrimination parameter* in IRT literature
  - orthogonal to *cutting line* in Nominate literature
- w_j *magnitude* of the jth hyperplane;
  - $w_j \boldsymbol{u}_j = \beta_j$, a discrimination parameter from the standard IRT model (Clinton, Jackman, and Rivers 2004)
- $\boldsymbol{x}_i$: multidimensional ideal point;
- α_j: difficulty parameter.

Prior:

$$\boldsymbol{x}_i \sim \mathcal{N}(0, \mathbb{I}), \quad \alpha_j \sim \mathcal{N}(0, \kappa), \quad \theta_{z_j} \sim \mathcal{N}(0, \mathbb{I}), \quad w_j \sim \mathcal{N}(0, \kappa) \quad (3)$$

Identification:

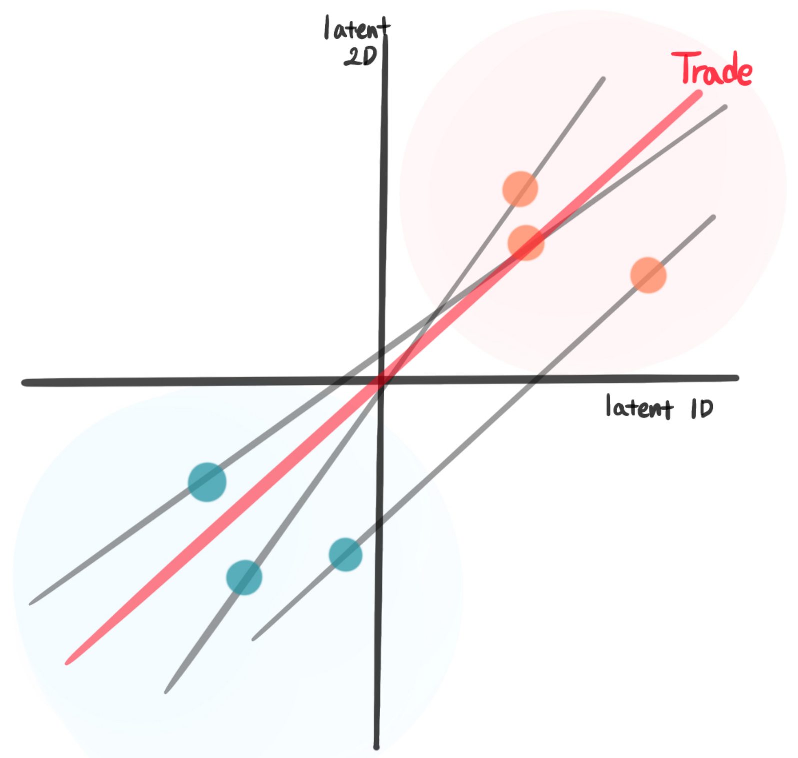
- Fix directionality of $\boldsymbol{u}_j$ to avoid reflection invariance
  - Code the voting data so that $y_{ij} = 1$ indicates a conservative vote
  - One may implement this by anchoring a conservative legislator

Issue-specific ideal points estimator:

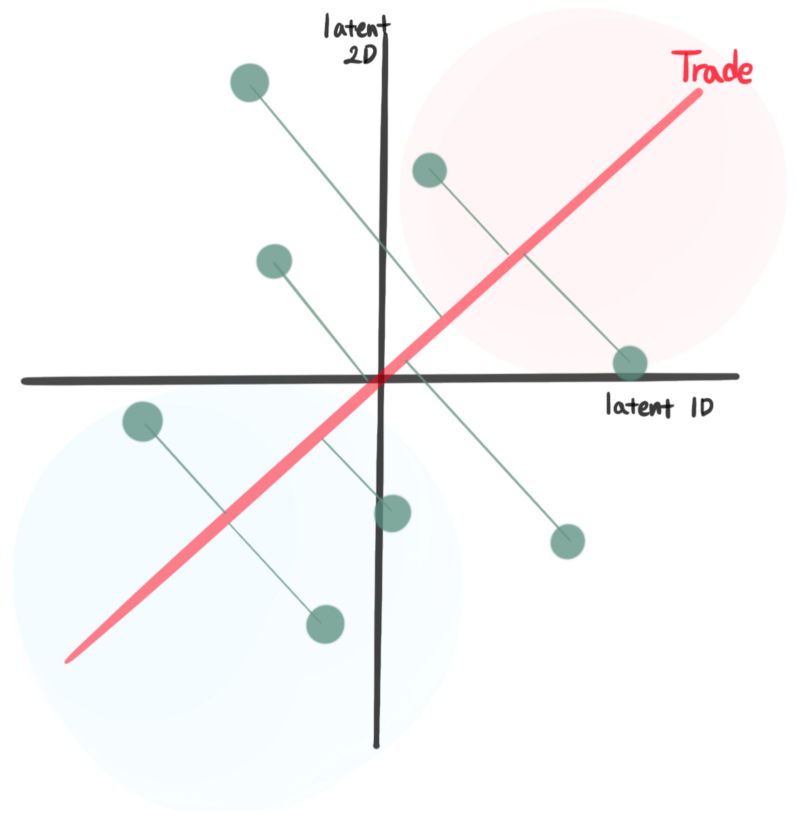
$$\hat{x}_i^*(z) := \frac{\hat{\theta}_z^\top \hat{\boldsymbol{x}}_i}{\hat{\theta}_z^\top \hat{\theta}_z} \quad (4)$$

Spatial Voting Theory

Geometric interpretation (2-dimensional example):



- Red dots: Yea positions (conservative votes)
- Blue dots: Nay positions (liberal votes)
- Grey lines: span($\boldsymbol{u}_j$) for $j \in$ {Trade bills}
- Red line: span(θ_{Trade}), a "Trade hyperplane"
  - Trade hyperplane is orthogonal to trade-specific decision boundary.



- In other words, legislators on the right side of this red line are conservative on trade issue and vice versa.
- Green dots: ideal points, $\boldsymbol{x}_i$
  - A projection of the ideal points on trade hyperplane yeilds trade-specific ideal points.

Properties of Issue-Specific Ideal Points

Proposition 1

$$\begin{aligned} y_{ij}^* &:= w_j \theta_{z_j}^\top x_i^*(z_j)\theta_{z_j} - \alpha_j & (5) \\ &= w_j \theta_{z_j}^\top \boldsymbol{x}_i - \alpha_j & (6) \\ &= \mathbb{E}_{\boldsymbol{u}_j}[\mathbf{1}\{y_{ij} = 1\}\}|\theta_{z_j}, w_j, \boldsymbol{x}_i, \alpha_j] & (7) \end{aligned}$$

- Issue-specific ideal points provide researchers a nice summary of voting behavior w.r.t. two randomnesses:
  - stochastic part of the utility,
  - deviation of the jth roll call's hyperplane($\boldsymbol{u}_j$) from the issue-specific hyperplane(θ_z)

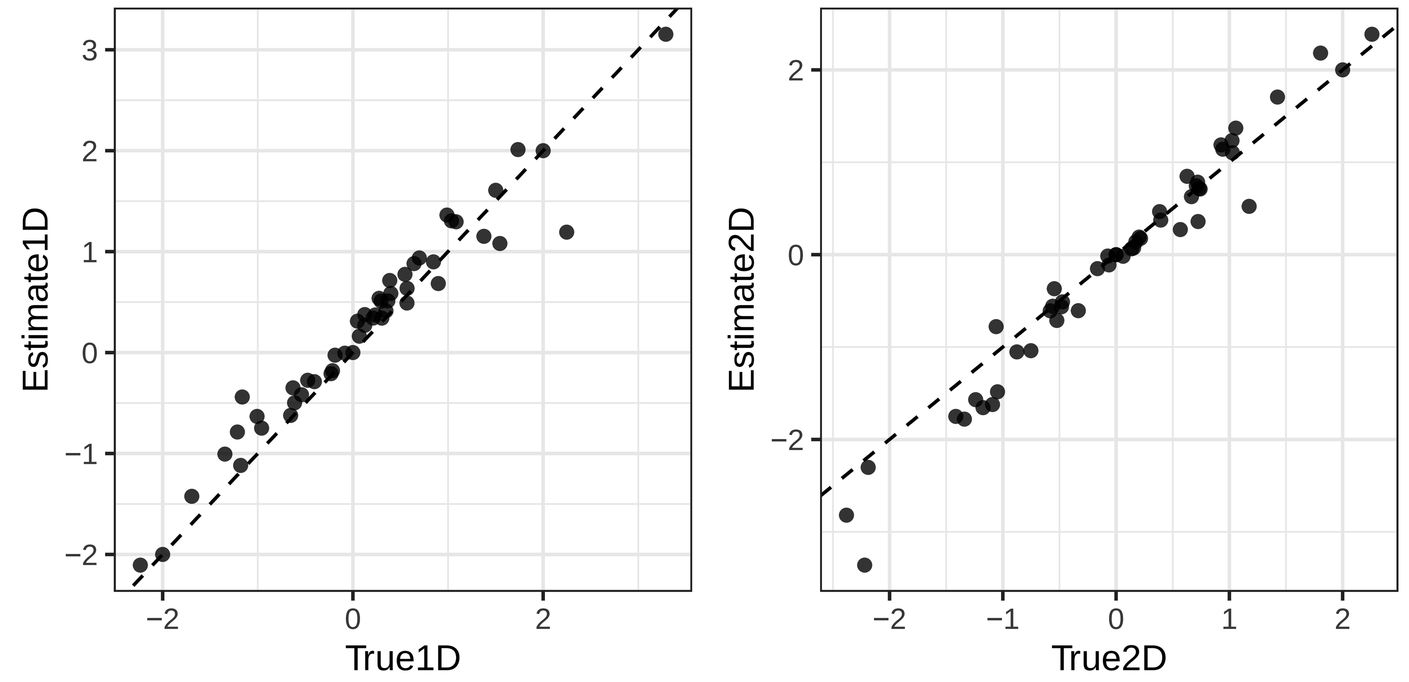
Proposition 2

$$\frac{\theta_z^\top \boldsymbol{x}_i}{\theta_z^\top \theta_z} = \frac{(R\theta_z)^\top R\boldsymbol{x}_i}{(R\theta_z)^\top R\theta_z} \text{ for } R^\top R = \mathbb{I} \quad (8)$$

- Unlike the multidimensional IRT model, issue-specific ideal points *do not* suffer from a rotational invariance.

Simulation Study

A simulation study with a synthetic data recovers true ideal points (50 legislators, 500 roll calls, 5 issue areas).



Contribution

Proposed Issue-Specific IRT model:

- Leverages information on the entire voting behaviors to map legislators on a hyperplane of the targeted issue;
- Issue-specific ideal points do not suffer from rotational invariance (i.e., do not require fixing $p + 1$ legislators in p dimensional policy space);
- Researchers can measure the similarity of different issue areas in a multidimensional policy space (e.g., $|cos(\theta_{\text{Trade}}, \theta_{\text{Immigration}})|$);
- Researchers can label latent dimensions using the knowledge of issue-specific hyperplane.

Next Steps

- More on simulation studies:
  - One-dimensional IRT v. proposed model;
  - Aggregate Proportional Reduction in Error (APRE) and Geometric Mean Probability (GMP) fit statistics.
- Application study:
  - US Congress
  - UN General Assembly
- Further stretch of the model: exploring a "bundle" of policies
  - How do policies on subject A entangle with policies on subject B? $\rightsquigarrow |cos(\theta_A, \theta_B)|$

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